Integration & Aggregation in Risk Management: An Insurance Perspective

Stephen Mildenhall
Aon Re Services

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Overview

- Similarities and Differences Between Risks
  - What is Risk?
  - Source-Based vs. Characteristic-Based Classification
  - Theoretical Tools
- Theoretical and Practical Challenges of Risk Integration
  - Dependencies
  - Modeling Philosophy & Guidelines
- Model “Insights” & Decision Making
  - What Can We Expect From a Model?

What is Risk?

- Risk: The Possibility Actual Differs From Expected
  - Balance Sheet Entries, Accruals, Valuations
  - Inadequate or Redundant or Both

- Three Characteristics of Risk
  - Severity
  - Time
  - Dependence

- Analysis/Synthesis Framework
  - Analyze Severity & Time Components Separately
  - Synthesis Requires Understanding of Dependence Between Risks

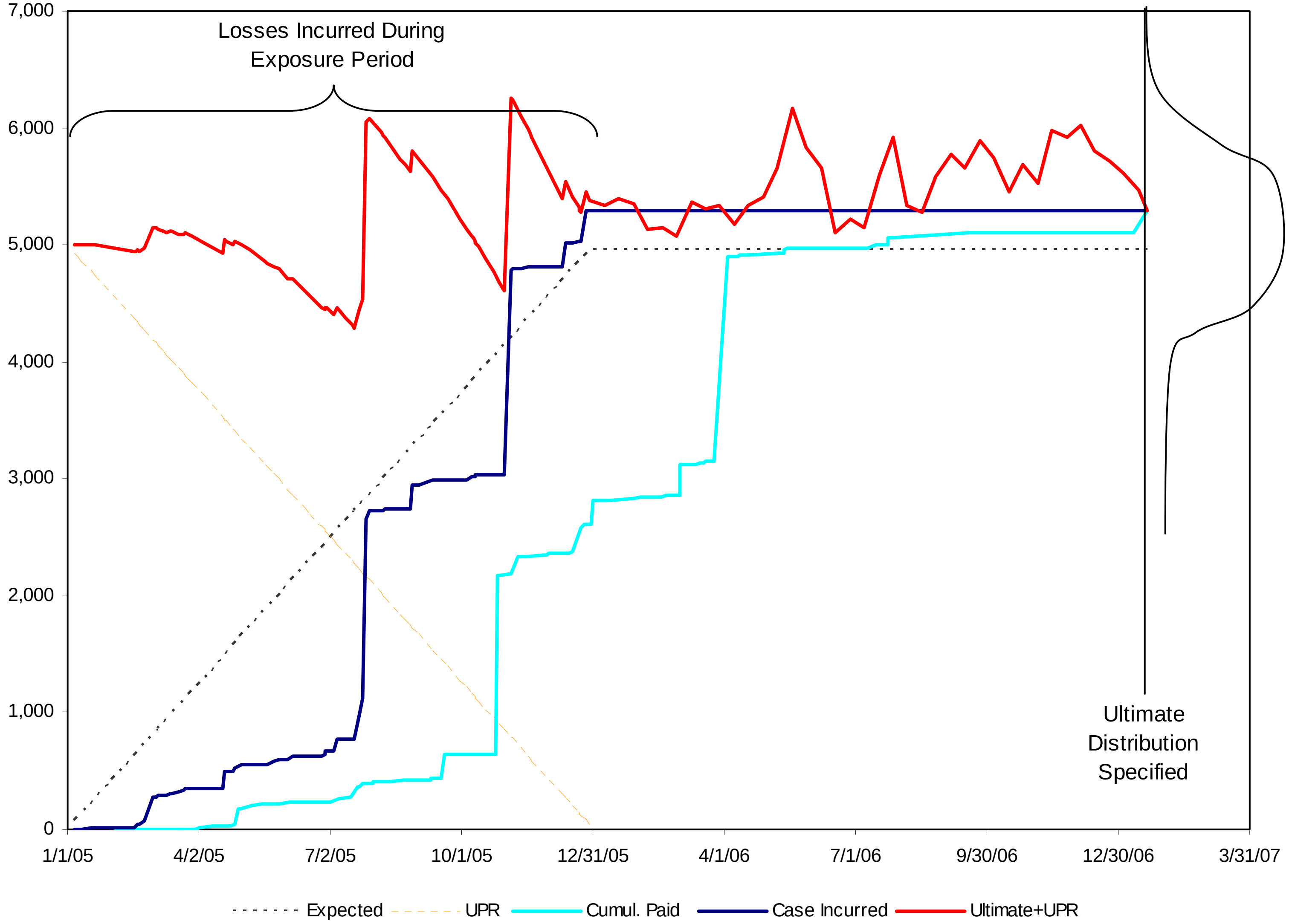
Classification of Risks

- Source-Based Classification (Practitioner)
  - Underwriting, Credit, Market, Liquidity, Operational
    - Developed Since 1990s in an Insurance Context
    - Lowe, Standard *Integrated DFA & Decision Support System,* 1996
    - Catastrophe Models, Early 1990s
- Characteristic-Based Classification (Academic)
  - **Severity** of Risk: Theory of Probability Distributions
    - Developed Since 1700s
    - Bernoulli, de Moivre, Laplace, Poisson, Gauss, Pareto
    - Extreme Value Theory, Thick-Tailed, Sub-Exponential, Distributions
  - **Time** Element: Stochastic Processes
    - Developed Intensively Since 1930s
    - Lévy, Khintchine, Kolmogorov, Doob, Meyer, Itô
    - Brownian Motion, Markov Processes, Lévy Processes
    - Critical to Development of Finance
  - **Dependence**: Statistical Association, Copulas
    - Newer Area of Research Since 1950s
    - Fréchet, Sklar

Time Characteristics of Risk

- Static View of Risk
  - P/C Actuaries Highly Trained in Static View of Risk
  - What is Distribution of AY Ultimate Loss?
- Dynamic View of Risk
  - ERM Requires Dynamic View of Risk
    - How Will Booked AY Ultimate Evolve Over Time?
    - Do Evaluations Between Statements Matter? (CP190, “must at all times”)
  - Theory of Stochastic Processes Highly Developed
    - Cornerstone of Modern Finance
  - Situation Vacant: Joint Stochastic Process Model
    - $(\text{Paid Loss, Case Incurred, Bulk Reserve})_t$
  - Bulk Reserve = *f* (Paid Loss, Case Reserve)
  - Simulation of Ultimate Loss Must Be Expanded To Simulation of Evolution of Paid Loss, Reserve & Ultimate Loss Over Time
    - Approach Crucial to Modeling Reserve Uncertainty

Time Characteristics of Risk

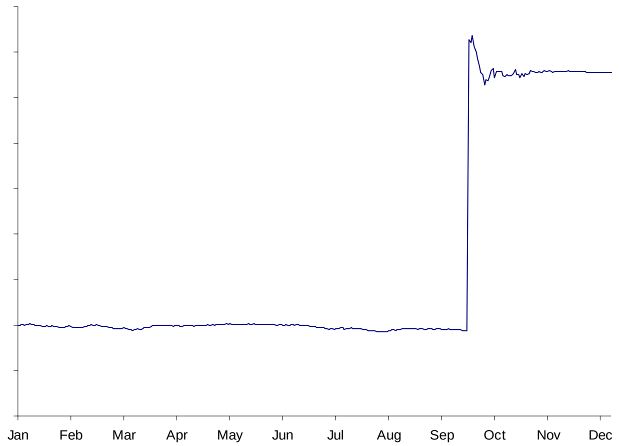


Time Characteristics of Risk

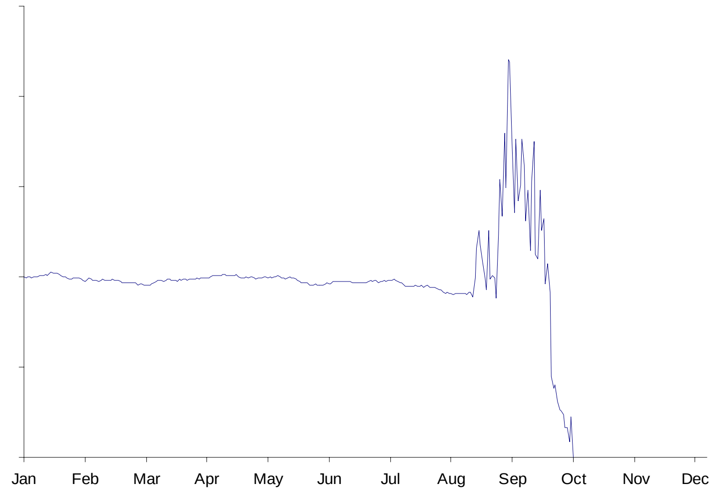
- Risk Can Evolve in Jumps or Continuously or Both
  - Price Evolution of Contract to Pay A Portion of US Hurricane Losses in Sept. 2005 vs. US Earthquake Losses in Sept. 2005

Hurricane Contract With Event

Earthquake Contract With Event



Hurricane Contract, No Event



Time Characteristics of Risk

- Two Basic Processes
  - Continuous Evolution: Brownian Motion
  - Jump Evolution: Poisson Process
- Aggregate Loss Model Gives Jump Process

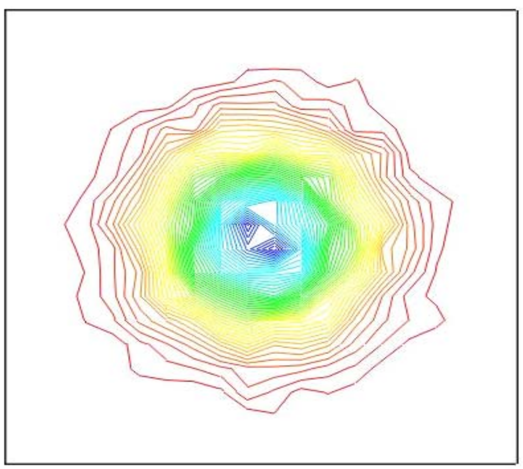
$$A = X_1 + \cdots + X_N$$

  - Frequency *N*, E(*N*)=Expected Counts Per Unit Time
  - *N* Often Poisson
  - Severity *X* From Usual Suspects
- Generalizing Aggregate Loss Model To Poisson Process
  - Define Frequency Density $\lambda(t)$ Which Can Vary Over Time
  - Expected Frequency Between 0 and *t* Given By $N(t) := \int_0^t \lambda(t)dt$
  - Actuaries Well Placed to Analyze & Model Risk Evolution

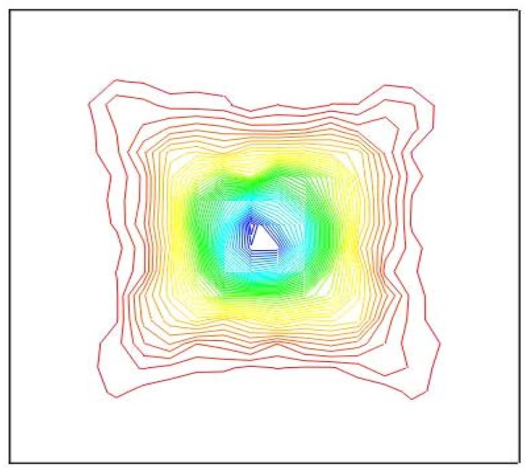
The Challenge of Risk Integration

- Next Step In Analysis/Synthesis Framework: Risk Integration
- The Challenge: **Dependence!**
- Long Term Capital Management
- Marginals & Correlation Structure Do Not Determine Distribution
  - Mean & Standard Deviation Do Not Determine Univariate Distribution

Normal Copula



***t*- Copula**



The Challenge of Risk Integration

- Structural Economic-Scenario Based Models
- Correlations & Dependencies Among All Risk Sources, CAS Working Party
  - Quasi-Structural Contagion Models (Glenn Meyers)
  - Bivariate Fourier Transform (David Homer)
  - Iman-Conover Method (SM)
  - Copulas
  - Reproduce Qualitative Behavior
    - Useful When Aggregate All That Matters
    - Use FFTs to Add Zero Mean "White Noise"

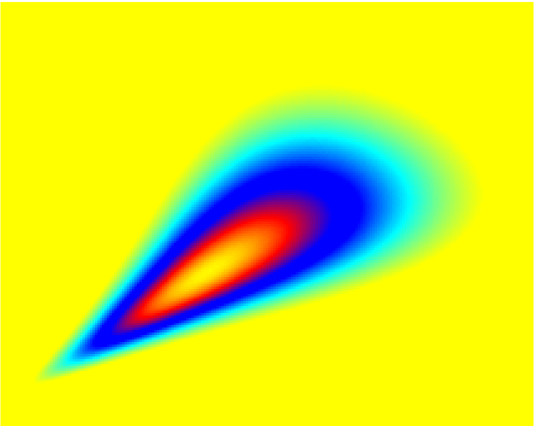
Iman-Conover Method

- Iman Conover (IC) Method
  - Given Input Sample from Desired Marginal Distributions
  - Re-order Sample to Have Same RANK ORDER as a Reference Multivariate Distribution With Desired Linear Correlation
- Method Effective Because
  - Rank and Linear Correlation Close
  - Easy to Produce Reference Multivariate Distributions
- IC Used By @Risk Software
- IC Algorithm, Inputs
  - Sample (n x r matrix) From Marginal Distributions
    - E.g. n ~ 10,000, r=2 for Bivariate Distribution
  - Correlation Matrix (r x r matrix)
- IC Algorithm, Output
  - Sample Re-ordered With Desired Correlation
- Reference Distributions Generated Using Choleski Trick
  - Elliptically Contoured Distributions (Normal, t, Laplace)

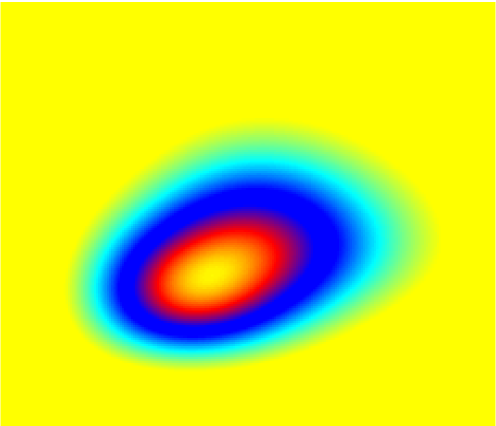
Copulas

- Copula: A Multivariate Distribution With Uniform Marginals
- Sklar's Theorem: Copulas Determine Multivariate Dependencies
  - $\Pr(X_1 < x_1, \ldots, X_n < x_n) = C(F_1(x_1), \ldots, F_n(x_n))$
- Copulas Generate Many Different Dependency Structures
- Simulating From Copulas Can Be Difficult
  - Archimedean Copulas Easy To Simulate From

Cook



FGM



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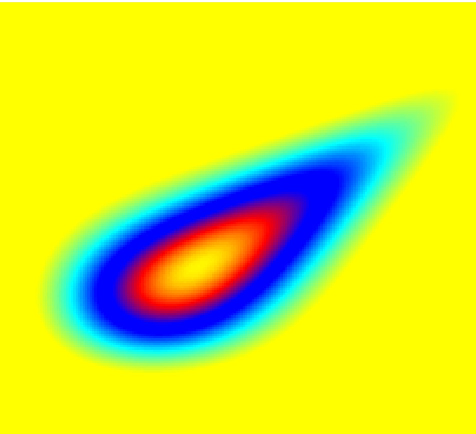


Modeling Philosophy & Guidelines

- Avoid Sweeping Generalizations
- Begin With The End In Mind
- Understand Process – *Then* Model
- Model Insights: Reasonable & Unreasonable Expectations

Avoid Sweeping Generalizations

- For Every Rule About Risk There Is A Counter-Example
- Pathological Examples
  - 99^{th} Percentile As Risk Adjusted Value
    - Any Percentile Can Be Less Than The Mean
    - Implies Negative Risk Load
  - Standard Deviation as Risk Measure
    - Pareto Can Have Same Mean & Lower SD Than a Uniform
  - Uncorrelated But Dependent
    - *t*-Copula vs. Normal Copula
- Be Aware of Limitations of Assumptions
- Intellectually Rigorous Framework Desirable
  - Coherent Measures of Risk

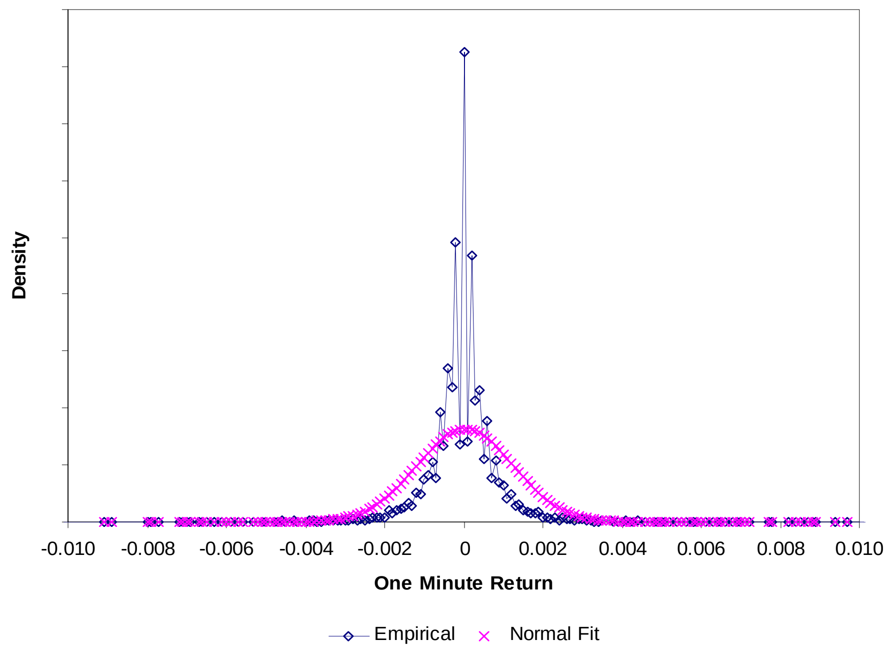
Begin With the End in Mind

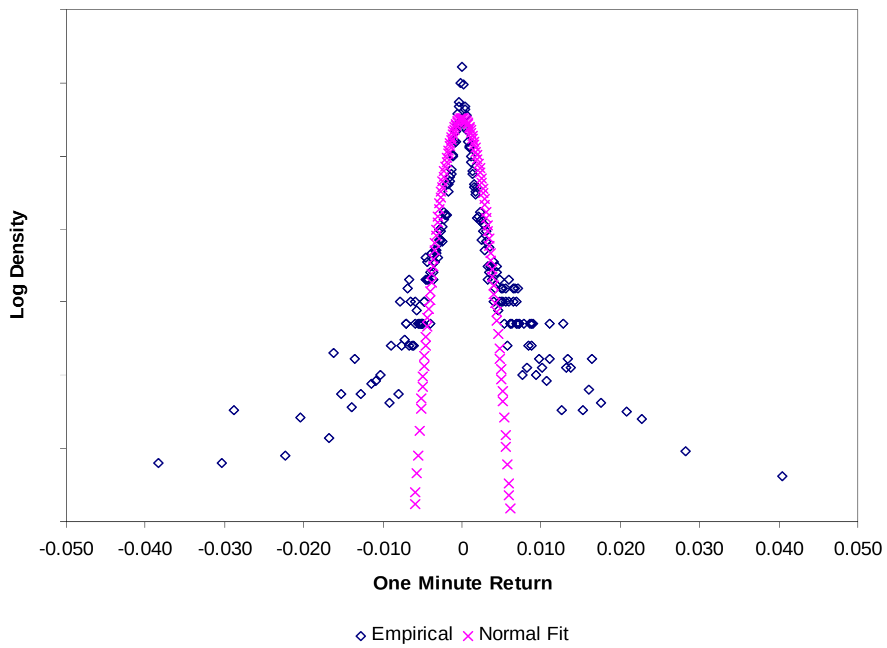
- Building An ERM Model Like “Building A Car”
  - Both Require Goal-Driven Design Objectives
- ERM Goals Include
  - Reinsurance Decisions
  - Capital Determination
  - Capital Allocation
  - Set BU Profit Targets
  - General Business Planning
  - Investment Opportunities
  - Acquisitions
  - Growth Strategy
  - Investment vs. UW Risk
  - Reserving & Capital

Understand Process – *Then* Model

- Don’t Let Modeling Expediencies Drive Model Process
- Workers Compensation Claim Payment Process
  - Driven By Mortality & Medical Cost Escalation Assumptions
  - Not Modeled Well Using Traditional P/C Actuarial Methods
  - Triangle Methods Ignore Changing Claimant Demographics
- Premium Correlation vs. Loss Correlation
  - Dependence in Results Driven By Premium Dependence
  - Catastrophe Losses Exhibit Quantifiable Loss Correlation
- Minimum Pension Liability
  - Difference of Asset & Liability Under Statutory Accounting
  - Very Sensitive To Investment Return Assumptions
- Example: Stock Price Returns

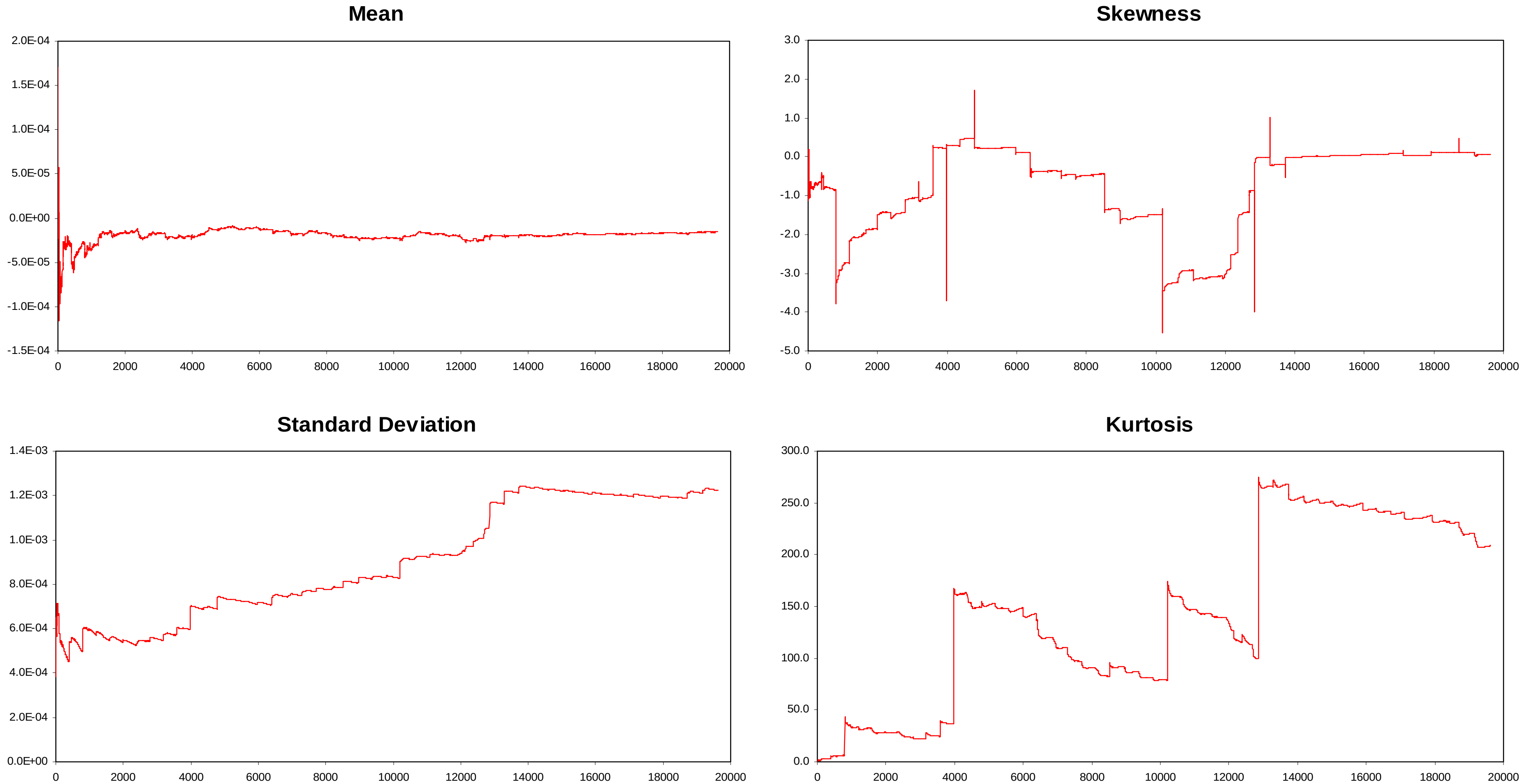
Example: Stock Prices





- Density of 1 Minute Returns Not Normally Distributed
- Largest Observed Changes ±4%
  - Most Big Moves Occurred Late In Trading Day, Between 15:10 and 15:20
  - For Normal Model ± 4% is a 1 in 10^{233} Event
  - Actually Occurred Twice in 19,000 Observations
- Is Difference in Distribution Important? Perhaps!

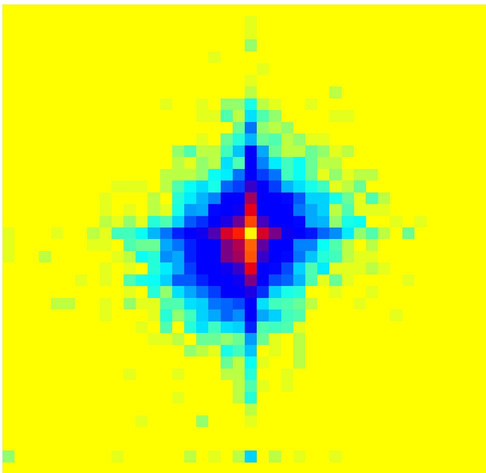
Example: Stock Prices

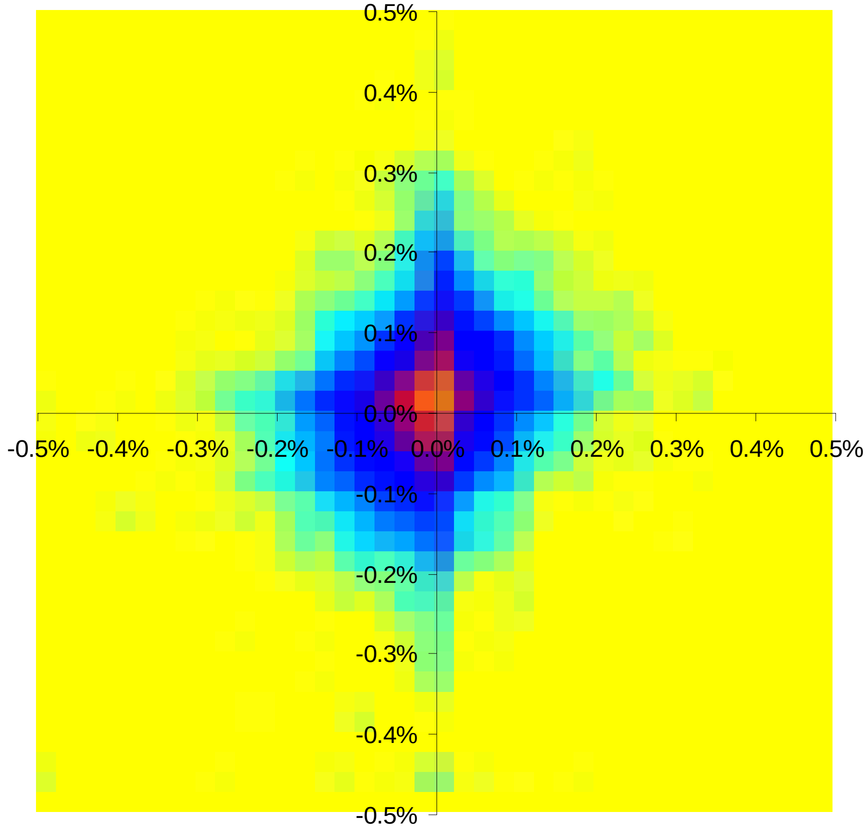


- Sequentially Computed Moments of 1 Minute Returns, Mandelbrot Converging Moment Test
  - F. Longin, *Asymptotic Distribution of Extreme Stock Market Returns,* J. of Bus., 1996 69(3)
  - Concluded First Two Moments Exist From 29,000 Daily Observations

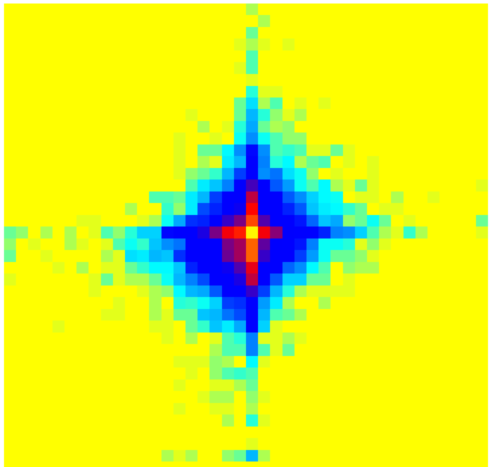
Example: Stock Prices

- Bivariate Distribution of 1 Minute Returns For Two Large Stock Companies, Feb-Apr 2005
- SD_1=0.075%, SD_2=0.103%
- Correlation 18.34%

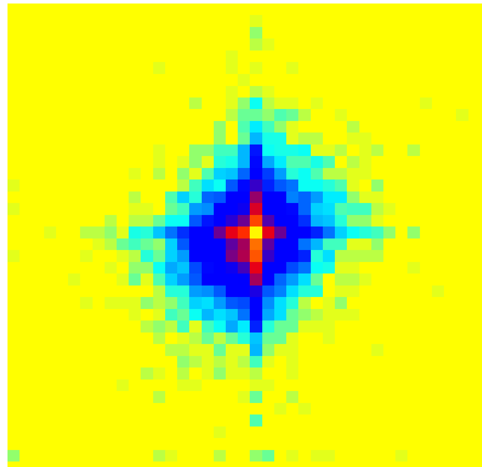




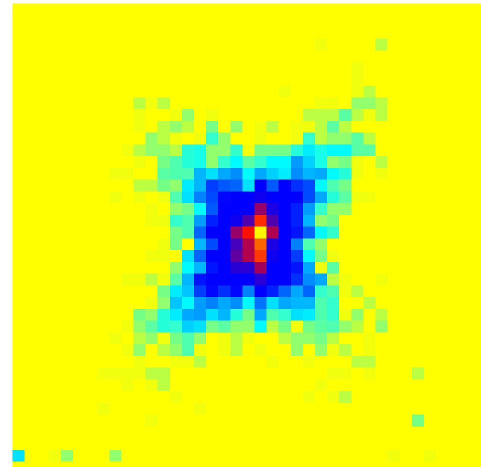
Example: Stock Prices, IC Method



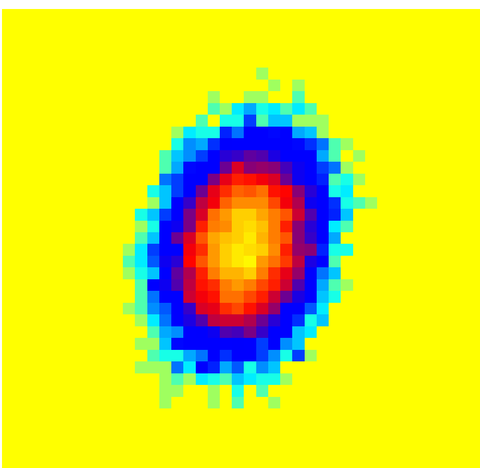
Actual Marginals, Normal Copula



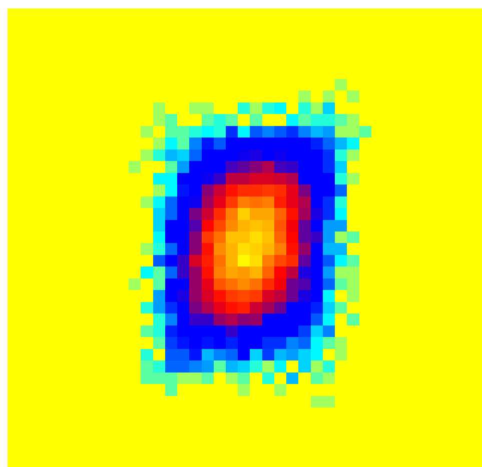
Actual Marginals, *t*-Copula, 5 DoF



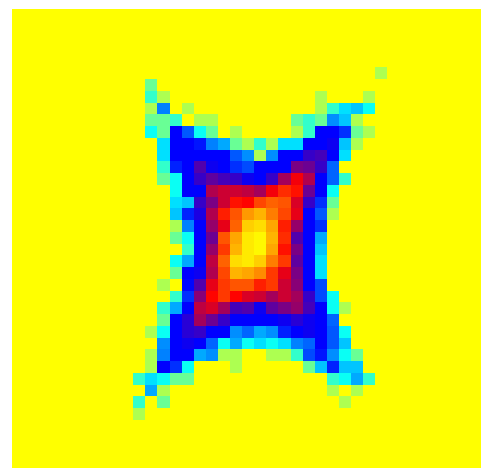
Actual Marginals, *t*-Copula, 1 DoF



Simulated Marginals, Normal Copula



Simulated Marginals, *t*-Copula, 5 DoF



Simulated Marginals, *t*-Copula, 1 DoF

Use of Model Results

- What Can We Expect From Models?
- Model Output Always Reflects Model Assumptions
- Management Reaction To Events & Feedback Loops Impossible to Model
- Reasonable Expectations
  - Reinsurance
    - Adequacy & Effectiveness
  - Capital
    - Determination & Allocation
  - Detailed Short-Term Calculations
    - Cash-Flow Projections
    - RBC, BCAR Projections
  - Growth Strategy
    - Adequate Income & Capital to Support Business Plan?
  - Stochastic Analysis of Static Plans
  - Weed Out Bad Strategic Options
- Unreasonable Expectations
  - Optimize ________
    - Management Role To Decide Between Efficient Choices
    - No Universal Evaluation Criteria
    - Model Can Provide Guidance
  - Investment Decisions
    - Parrot Assumptions
    - Assumptions Article of Faith
    - Tony Day, *Financial Economics & Actuarial Practice*, NAAJ 8(3)

Summary

- Actuarial Analysis of Severity Well Developed
- Theory of Time Evolution of Risk Available & Readily Comprehensible to Actuaries
- Theory of Risk Dependencies Still Under Development
- Model With Goal in Mind
- Question Model Insights; Apply With Caution